Iterative optical processor (IOP) for adaptive phased array radar processing

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ABSTRACT

A non-coherent vector-matrix multiplication system (using a linear LED input array, a covariance matrix mask and a linear photodiode detector array) is described. The system has been modified to perform complex multiplications and by feedback to be an iterative optical processor (IOP) to solve the adaptive phased array radar processing problem.

1. Introduction

The real-time and parallel processing advantages and features of optical data processing that have intrigued researchers for nearly two decades since the advent of the laser have not been fully exploited to produce powerful deliverable systems. Recent advances in solid state technology (advances in photodiode detector arrays fostered by production line inspection systems, etc.) have led to a new generation of optical processors that are compact and easily engineered to meet in-field operational system requirements. The specific optical computer we describe performs a matrix inversion in parallel and real-time. The basic components of this system are: an input linear LED array, a mask, and a parallel readout linear photodiode detector array. The conceptual foundations for such optical matrix multiplications were laid by L. Cutrona [9] in 1965. An extensive study of possible implementations of such systems was conducted in 1966 by researchers at General Electric [1]. An iterative version of such a system was also suggested [10] in 1966. Since then, an impressive engineered electro-optic matrix multiplier has been demonstrated by Keith Bromley and his co-workers at NOSC [2]. More recently, J.W. Goodman and his group at Stanford have extended these techniques by the use of a linear LED input array [3]. In this paper, we combine many of the above techniques as well as others for an adaptive phased array radar problem.

Signal processing represents a major application in which the real-time and parallel processing features of optical systems can be fully utilized and in which the application requirements for the real-time processing of data of high bandwidths exists. Radar signal processing is one application area in particular to which optical processing has been extensively applied [4]. One of the more advanced concepts in present radar systems is the adaptive phased array [5-7]. These systems present even more pressing requirements on the associated data processor than do more conventional radar systems. This adaptive phased array radar application is thus most appropriate for an optical signal processing solution.

In Section 2, we describe the adaptive phased array radar signal processing problem. Both a matrix inversion and an iterative formulation of the required processing are included. In Section 3, an iterative electro-optical matrix multiplier processor system is described. In Section 4, we demonstrate how this optical system can solve the adaptive phased array radar problem. In this section, we also describe an extension of this system that enables it to operate on complex data. The specific application of this system to the adaptive phased array radar problem with complex data values is then discussed. Experimental verification of our system in this adaptive phased array radar application is included in Section 5.

2. Adaptive Phased Array Radar Signal Processing

A phased array radar system can be described most simply by considering the 1-D phased array radar (transmitting or receiving) system shown in Fig. 1. Beam steering in such systems is achieved by varying the weights W_n applied to each element n of the array. If the same weight is applied to all elements, the beam is steered on-axis or at boresight. The resultant beam pattern is described by

$$E(\theta) = K\,\frac{\sin[(\pi N d/\lambda)\sin\theta]}{\sin[(\pi d/\lambda)\sin\theta]} , \tag{1}$$

where d is the spacing between each of the N phased array elements. The pattern described by Eq. (1) is the classic sinc x = [sin (x)]/x pattern. If the phases ϕ_n of the weights W_n applied across the array elements increase linearly across the array, the resultant beam pattern from the array will be steered to $\theta = \theta_o$ where θ_o is proportional to the phase gradient across the array. The array has higher gain in the direction of the beam than it has at all other directions. These other directions constitute the sidelobes of the array pattern. When properly designed the radar will respond to signals backscattered from targets located within the directed beam while being relatively insensitive to objects and other signal sources within the sidelobes. This enables the classification of received energy as signals or noise depending on whether it is present in the main beam or in the sidelobes respectively. When the noise energy is much higher than the signal energy, the radar system becomes unable to discern the target and performance is deteriorated. However, if the sidelobe response can be altered in the direction of such noise sources, system performance can be recovered. In order to alter the array factor to this end, it is necessary to augment the phase and/or amplitude weight of each element so that the pattern has maximum gain in the direction of the steered beam and minimum response in the sidelobe region corresponding to noise sources. In an adaptive phased array, the weights at each element are chosen to place nulls at the angular locations of interfering noise power incident at the antenna. This provides a great increase in detectable SNR. The adaptive weights W_n vary with time and are functions of the time-varying external noise field. To determine W_n, the expected values or time averages of the products of all pairs of signals V_i and V_j received at array elements i and j are determined. The co-variance matrix M with components

$$M_{ij} = \int V_i(t) V_j^*(t) dt \tag{2}$$

is then formed. The weights W in vector notation are related to the co-variance matrix M and the steering vector S by

$$MW = S^*.$$

The desired weights W for the N array elements can then be obtained from S^* and the inverse M^{-1} of the co-variance matrix as described by

$$W = M^{-1} S^*. \tag{4}$$

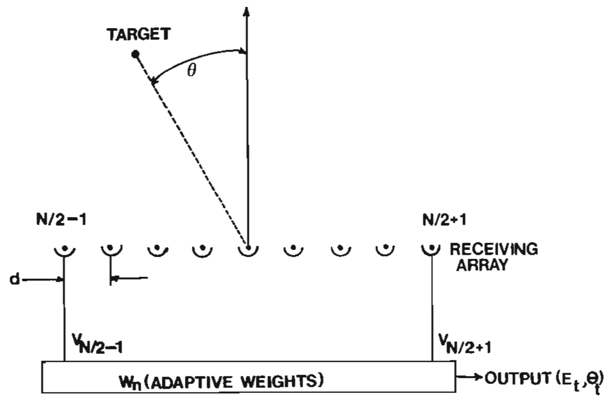


Fig. 1 Phased array radar receiving scenario and notation.

The major computational load associated with determining the weights W for an adaptive phased array radar is the matrix inversion operation. For large arrays, M is quite large and its inversion is very demanding in terms of the number of operations required, the computational time involved, and the memory storage requirements associated with these operations. In Section 3, we describe a noncoherent iterative optical processor (IOP) that can determine W given M and S using the iterative algorithm

$$W_{i+1} = S^* + (I-M)W_i, \tag{5}$$

where I is the identity matrix and W_i is the estimate of the adaptive weights after iteration i. When $W_i = W_{i+1}$, the processor has converged.

3. Iterative Optical Processor

The full iterative optical processor (IOP) is shown schematically in Fig. 2. We first describe this system as a matrix vector multiplier and then extend the analysis to the case of a matrix inversion iterative optical system for adaptive phased array radar. For this initial discussion, we ignore the output adder and the feedback system shown in Fig. 2. The input plane P_1 contains a linear array of light emitting diodes (LEDs). A lens system (omitted from Fig. 2 for clarity) spreads the light output from each LED into a long thin horizontal pencil beam. With N input LEDs and N horizontal rows on the mask at P_2, the light from the nth LED passes through the nth row of the mask in P_2. If we describe the irradiance of the nth LED input as a_n and the transmittance of the mask by b_{nm}, then the light distribution leaving row n of P_2 is

$$a_n b_{n1} + a_n b_{n2} + \ldots + a_n b_{nM}. \tag{6}$$

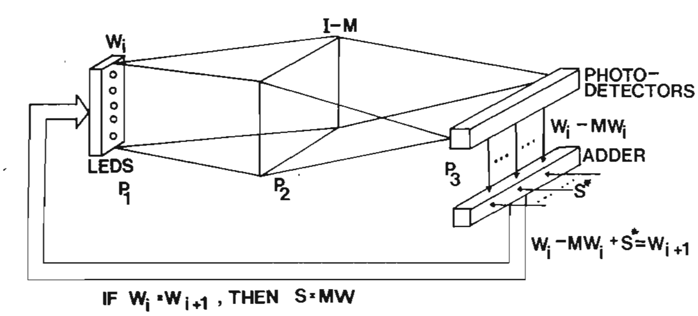


Fig. 2 Schematic diagram of an iterative optical processor for adaptive phased array radar processing.

A second lens system follows P_2 (it is likewise omitted from Fig. 2 for clarity). This lens system images P_2 horizontally onto a linear array of M detectors. In the vertical direction, it focuses the light from each column m in P_2 onto output detector m. Thus, at output detector m, the detected signal is proportional to the summation of the light transmitted through all N horizontal channels or rows in the mth column of the mask at P_2. This signal is of the form

$$c_m = \sum_{n=1}^{N} a_n b_{nm} = a_1 b_{1m} + a_2 b_{2m} + \dots + a_N b_{Nm}. \tag{7}$$

This resultant vector C is the desired product of the vector A and the matrix B and the system thus forms

$$[C_m] = [A_n]\ [B_{nm}]. \tag{8}$$

The basic noncoherent matrix vector multiplication system was described earlier for use as a discrete optical system to realize an FFT operation [3]. Prior optical matrix multiplication systems using single LED inputs [2] and a 1-D input function in real-time [1] have been described and considered as early as 1966. We now extend the operation of this system as an iterative optical processor for adaptive phased array radar processing, specifically for computation of the necessary time varying adaptive weights W_n. The iterative algorithm in Eq. (5) is used. In this case, the N electronic inputs to the LEDs at cycle i are $[W_n]_i$ and the 2-D matrix mask at P_2 is $[I-M_{nm}]$, where I is the identity matrix and M is the co-variance matrix defined in Eq. (3). The system's outputs from the photodetectors at P_3 are then $[W_n]_i[I-M]$. This vector output is added to the steering vector S^* in the adder shown. The adder's output is then

$$[W\ ]_{i+1} = [I-M]\ [W]_i + S^*. \tag{9}$$

In Eqs. (9) and (5), we denote the system's output at the ith iteration by $[W\ ]_{i+1}$ and W_{i+1}, respectively. As Eqs. (5) and (9) show, with W_i as the input, the system's output is W_{i+1} (the value of W after the ith iteration).

Thus, by cycling the above system a sufficient number of times for W to converge to its final value, this IOP does in fact solve Eq. (3) for W. With 1 μsec cycle times possible from the LED inputs to the adder outputs, total processing times of only 0.1 to 1.0 msec or less are required to determine the adaptive weights for a 100 or 1000 element adaptive phased array radar.

4. Complex Matrix Multiplication for Adaptive Phased Array Radar

We now direct attention to the mask [I-M] in Fig. 2, with specific attention to the complex values required for its elements and to how complex matrix multiplication (as is required for adaptive phased array processing) can be realized in this IOP system. This is best seen by a simple example. We consider a two element array (with a spacing d between elements). With one external noise source at an angle θ_1 with respect to the normal to the array, the signals received at the two array elements are then

$$v_1 = x_1 + y_1 \tag{10a}$$

$$v_2 = x_2 + y_2 \quad , \tag{10b}$$

where x_i is the interference voltage in channel i and y_i is the noise voltage in channel i. The voltage x_2 will lag x_1 by a phase angle given by $\gamma = (2\pi d/\lambda)\sin\theta_1$, i.e. $x_2 = x_1 \exp(-j\gamma)$, and the noise voltages y_1 and y_2 will be independent of each other and of the interference signals. We denote the noise power in each element channel due to this external noise source by P and the receiver noise power in each channel by N. For this case,

$$M = \begin{bmatrix} P + N & P\exp(-j\gamma) \\ P\exp(+j\gamma) & P + N \end{bmatrix} . \tag{11}$$

For the experimental demonstration to follow and in the interest of analyzing the specific scenario, we chose a noise source emitting P = 0.1 watts of power at an angle θ_i such that $(2\pi d/\lambda)\sin\theta_i = \pi/3$. We also assume a receiver noise power N = 0.5 (5 times larger than the received power P due to the noise source itself). This latter N/P = 5 ratio was chosen because it results in a faster convergence rate for the iterative processor. For this case, the actual matrix mask used becomes

$$[I-M] = \begin{bmatrix} 1-P-N & P\exp[-j(\pi/3+\pi)] \\ P\exp[+j(\pi/3+\pi)] & 1-P-N \end{bmatrix} . \tag{12}$$

The components of this matrix are complex, whereas the transmittance of the areas of the mask must all be real and positive. To realize complex matrix multiplication, we represent a complex number m' (one component of the matrix [I-M]) by three components (m_0', m_1', and m_2'), the projections of m' along the axes at angles of 0°, 120°, and 240° in complex space. As described by Goodman, et al. [8], we represent m' by

$$m' = m_0' \exp(j0) + m_1' \exp(j2\pi/3) + m_2' \exp(j4\pi/3), \tag{13}$$

where m_0', m_1' and m_2' are all real and positive numbers. For the example chosen, the phase angles of the four components of [I-M] are 0°, $-(\pi/3+\pi) = -240° = +120°$, and $+(\pi/3+\pi) = +240°$, respectively. Thus, the three components of [I-M] for our example become

$$M_0' = [I-M]_{\hat{0}} = \begin{bmatrix} 1-P-N & 0 \\ 0 & 1-P-N \end{bmatrix} , \tag{14a}$$

$$M_1' = [I-M]_{\hat{1}} = \begin{bmatrix} 0 & P \\ 0 & 0 \end{bmatrix} , \tag{14b}$$

$$M_2' = [I-M]_{\hat{2}} = \begin{bmatrix} 0 & 0 \\ P & 0 \end{bmatrix} . \tag{14c}$$

$$\begin{bmatrix} m_0' & m_1' & m_2' \\ m_2' & m_0' & m_1' \\ m_1' & m_2' & m_0' \end{bmatrix}$$

(a)

$$\left[\begin{array}{cc|cc|cc} 0.4 & 0 & 0 & 0 & 0 & 0.1 \\ 0 & 0.4 & 0.1 & 0 & 0 & 0 \\ \hline 0 & 0.1 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0.1 & 0 \\ \hline 0 & 0 & 0 & 0.1 & 0.4 & 0 \\ 0.1 & 0 & 0 & 0 & 0 & 0.4 \end{array}\right]$$

(b)

Fig. 3 The mask [I-M] for complex matrix multiplication.
(a) General form, (b) Specific form for our scenario.

A general formulation for the [I-M] matrix in terms of these m_i' components is shown in Fig. 3a. The specific form of the [I-M] mask in the system of Fig. 2 for our 2 element phased array example is shown in Fig. 3b. To produce the complex vector matrix multiplication we use three input LEDs to represent each complex W_n input value. Thus, for the two element phased array radar example, six input LEDs are used. We describe the two complex input numbers W_n for the two array elements at iteration i by $W_i(n)$ or more specifically by $W_i(1)$ and $W_i(2)$. For iteration i+1, we describe these two complex input numbers by $W_{i+1}(1)$ and $W_{i+1}(2)$. In terms of their three real and positive component numbers at phase angles of 0°, 120°, and 240° in the complex plane) used to represent any W_i, we describe W_i and W_{i+1} as

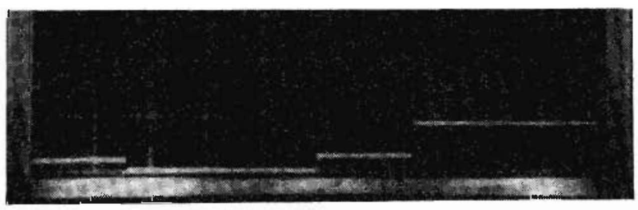
$$\vec{W}_i = \left(W_{i,\hat{0}}(1),\ W_{i,\hat{0}}(2),\ W_{i,\hat{1}}(1),\ w_{i,\hat{1}}(2),\ W_{i,\hat{2}}(1),\ W_{i,\hat{2}}(2)\right) \tag{15a}$$

$$\vec{W}_{i+1} = \left(W_{i+1,\hat{0}}(1),\ W_{i+1,\hat{0}}(2),\ W_{i+1,\hat{1}}(1),\ W_{i+1,\hat{1}}(2),\ W_{i+1,\hat{2}}(1),\ W_{i+1,\hat{2}}(2)\right) \tag{15b}$$

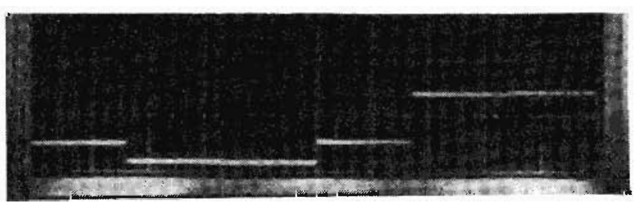
With the 6 LED inputs arranged as described in Eq. (15a) and the mask [I-M] arranged as shown in Fig. 3, the outputs at the 6 photodiode detectors (after addition of S^*) are as described in Eq. (15b). These outputs can then be directly fed back to the LED inputs as in Fig. 2.

TABLE 1 Theoretical W_{i+1} Outputs from the IOP of Fig. 2 at Iteration i

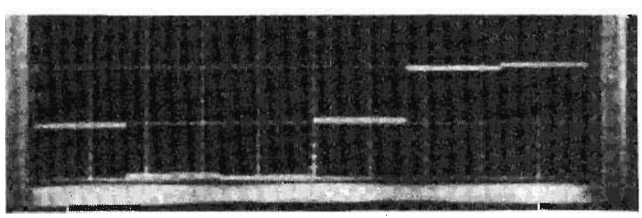
	$W_{i+1,\hat{0}}(1)$	$W_{i+1,\hat{0}}(2)$	$W_{i+1,\hat{1}}(1)$	$W_{i+1,\hat{1}}(2)$	$W_{i+1,\hat{2}}(1)$	$W_{i+1,\hat{2}}(2)$
0	0.3	0.3	0	0	0	0
1	0.42	0.42	0.03	0	0	0.03
2	0.471	0.471	0.054	0	0	0.054
3	0.4938	0.4938	0.0687	0	0	0.0687
4	0.5043	0.5043	0.0768	0	0	0.0768
5	0.5094	0.5094	0.0811	0	0	0.0811
6	0.5118	0.5118	0.0834	0	0	0.0834
∞	0.5142	0.5142	0.0857	0	0	0.0857



(a) i = 0



(b) i = 1



(c) i = 5

Fig. 4 Outputs from the six photodiode detector elements in the IOP after iterations 0, 1, and 5. Comparisons to the data in Table 1 shows the excellent agreement and accuracy of this IOP system.

5. Experimental Demonstration

Solving Eq. (5) for the adaptive weights we obtain

$$[W] = \frac{C}{(P+N)^2 - P^2} \begin{bmatrix} P + N-P \exp(-j\gamma) \\ \\ P + N-P \exp(+j\gamma) \end{bmatrix} . \quad (16)$$

We choose C = 0.3, where the steering vector is

$$\left[S^*\right] = \begin{bmatrix} C \\ \\ C \end{bmatrix} . \quad (17)$$

This choice of C was selected to make the multiplication factor in Eq. (16) approximately equal to unity (to simplify the associated mathematical analysis). Evaluating Eq. (16) for our numerical example, we find

$$W = \frac{0.3}{0.35} \begin{bmatrix} 0.6 - 0.1 \exp(-j\pi/3) \\ \\ 0.6 - 0.1 \exp(+j\pi/3) \end{bmatrix} . \quad (18)$$

The results of a computer simulation for the 6 components of $[W_{i+1}]$ in Eq. (16) after each of the first 6 iterations (i=0 through i=6) are given in Table 1. The outputs from the 6th iteration through the system are quite close to the final exact values predicted in Eq. (18). The system of Fig. 2 using the mask described in Fig. 3 and the input format described in Eq. (15a) was assembled. The outputs from the 6 photodetectors in the IOP system are shown in Fig. 4 after the 0, first, and fifth iteration through the system. Comparing these values to the ones noted in Table 1, we see that the IOP system performs the required operations within reasonable accuracy.

6. Summary and Conclusion

In summary, an IOP matrix multiplication system capable of operating on complex matrix elements has been described for the specific application of an adaptive phased array radar processor. The general theory of the system and its experimental confirmation for a 2 element adaptive phased array have been shown. The system's resultant accuracy and outputs obtained were seen to agree with the theoretically predicted ones. This system is capable of operating at a minimum of 1 μsec cycle times. For a phased array with 1000 adaptive elements, a system as in Fig. 2 with 1000 LEDs, is capable of performing all of the necessary adaptive processing in less than a worst case processing time of 1 msec.

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