Fusing Images and Maps
Mark J. Carlotto, General Dynamics Advanced Information System

Spatial data sharpening techniques that fuse images and maps are described. The statistical basis of these techniques are reviewed and extended for sharpening other kinds of spatial data that can be difficult to collect in denied areas. One example is demographic data. We demonstrate the ability to derive high-resolution population maps from county or district census data and Landsat imagery that is accurate to within 5% of the true population within a test area.

Key words: Image-map fusion, spatial sharpening, Bayesian inference, population estimation, Landsat classification.

1. Introduction
Maps describe the spatial distribution of a variable or feature over some area. Examples include land use/land cover, elevation/slope, trafficability/mobility, demographics, etc. Depending on the imaging modality (e.g., radar, multispectral, etc.) coregistered near time-coincident imagery over the same area may be correlated with certain features in maps. In such cases, it is possible to use the map to classify the image, or equivalently to use the image to sharpen the map.

Statistical techniques have been developed for sharpening categorical (land use/land cover maps) and numerical (elevation/slope maps) data sets. Previously we have demonstrated the ability to use 25 m/pixel resolution Landsat TM to sharpen 1:250,000-scale land use/land cover maps, and to spatially enhance 1 km resolution digital elevation models to 25 m/pixel. The theoretical basis of these techniques are reviewed and extended for sharpening other kinds of spatial data that can be difficult to collect in denied areas.

2. Previous work
Previously, we have demonstrated the ability to use higher-resolution imagery to sharpen both lower-resolution categorical (thematic) and numerically valued feature maps. The map-based classifier [1] uses an existing land-use/land-cover map \( m(i,j) \) as a source of ‘truth’ information for classifying a coregistered, higher-resolution multispectral image

---

1 mark.carlotto@gd-ais.com
\( x(i, j) \). The classified image is effectively a spatially sharpened version of the original map

\[
\{ m(i, j), x(i, j) \} \rightarrow m'(i, j) \tag{1}
\]

where \( M \) is the number of classes. Unsupervised clustering (e.g., by K-means, vector quantization, or other means) converts the image \( x(i, j) \) into a cluster image \( k(i, j) \) of \( K \) unique clusters, where the number of clusters depends on the spectral complexity of the scene. The joint distribution \( p(k, m) \) computed from the coregistered cluster-class map, is the probability that the \( k \)-th image cluster occurs together with the \( m \)-th map class over the data set. Bayes rule assigns the class with the highest posterior probability to each cluster

\[
m'(k) = \underset{m}{\arg \max} \{ p(m | k) \}
\]

\[
= \underset{m}{\arg \max} \left\{ \frac{p(k | m) \hat{p}(m)}{p(k)} \right\} = \underset{m}{\arg \max} \left\{ \frac{p(k | m) \hat{p}(m)}{p(k)} \right\} \tag{2}
\]

The conditional distribution \( p(k | m) = p(k, m) / p(m) \) and priors \( \hat{p}(m) \) can be computed in several different ways as discussed in [1].

A different approach is required for numerically valued data. The sharpening process combines a low-resolution numerically valued feature map \( y(i, j) \) with a high-resolution image to produce a high-resolution feature map

\[
\{ y(i, j), x(i, j) \} \rightarrow y'(i, j) \tag{3}
\]

Bayes rules for categorical data minimizes the probability of error. For numerical data, we use the conditional expected value, which minimizes the mean-squared error of the estimate [2]

\[
y' = \sum_y y p(y | x) = \sum_y y p(y | k) \tag{4}
\]

The conditional mean is computed over all unique combinations of input image values. To reduce computation, this can be done instead over clusters, which gives an approximate value for the conditional mean. The elevation sharpening process [3] uses a higher-resolution multispectral image to sharpen directional-gradient information derived from a lower-resolution digital elevation model, which is then integrated into a higher-resolution DEM.
3. Sharpening Population Data

The above methods do not explicitly constrain the sharpened data. Consider a different kind of sharpener

$$\{y_0, y(i,j), m(i,j)\} \rightarrow y'(i,j)$$  

(5)

where the original and sharpened maps have to satisfy a constraint:

$$\sum_{i,j \in A} y(i,j) = \sum_{i,j \in A} y'(i,j) = y_0$$  

(6)

Specifically, we are interested in sharpening a lower-resolution population map (e.g., county census data) $y(i,j)$ using a higher-resolution thematic map such as a land-use/land cover (LU/LC) map $m(i,j)$, where $y_0$ is the total population over the area, $A$. Assume $y(i,j)$ contains $N$ census tracts, where $y(n)$ is the population density in the $n$-th tract. We wish to determine $y'(m)$, the population density corresponding to the $m$-th map class. The joint distribution $p(m,n)$ is the spatial overlap (co-occurrence) between the $n$-th census tract and the $m$-th map class. The fraction of the $m$-th map class in the $n$-th tract

$$f(m,n) = p(m \mid n) = p(m,n) / \sum_m p(m,n) = p(m,n) / q(n)$$  

(7)

where $q(n)$ is the relative size (frequency) of the $n$-th tract. The population density assigned to the $m$-th map class is the solution to

$$y(n) = \sum_m y'(m) f(m,n)$$  

(8)

subject to the constraint (6).

Consider a simple example consisting of sample population data in $N=3$ census tracts:

$$\begin{array}{|c|c|c|}
\hline
\text{20/0.4} & \text{10/0.3} & \text{100/0.3} \\
\text{33} & \text{33} & \text{33} \\
\hline
\end{array}$$

$$\begin{array}{|c|c|c|}
\hline
3 & 1 & 2 \\
\hline
\end{array}$$

$$q(n) = \begin{bmatrix} 0.3 \\ 0.3 \\ 0.4 \end{bmatrix}$$
The total population is 20+10+100=130. Now consider a coregistered map consisting of $M=3$ LU/LC classes:

$\begin{array}{cc}
1 & 2 \\
3 & &
\end{array}$

$p(m) = \begin{bmatrix} 0.5 \\ 0.4 \\ 0.1 \end{bmatrix}$

where $p(m)$ is the relative frequency of the $m$-th map class. The co-occurrence matrix and fractions are determined by intersecting the census tracts with the LU/LC map:

$p(m,n) = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0 \end{bmatrix}$

$f(m,n) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.67 & 0.33 \\ 0.5 & 0.5 & 0 \end{bmatrix}$

Writing (8) as $y = F y'$, the solution by matrix inversion is

$y' = F^{-1} y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .67 & .33 \\ .5 & .5 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 33 \\ 333 \\ 333 \end{bmatrix} = \begin{bmatrix} 33 \\ 67 \\ 875 \end{bmatrix}$

(9)

The estimated population over the map classes is 16.5+26.8+87.5=130.8, which is approximately what we started with.

In general the number of tracts will not be equal to the number of map classes. If the number of tracts is less than the number of map classes, then there may be multiple
solutions; or, if the number of tracts is greater than the number of map classes then there may be no unique solution. In general a constrained least-squares solution is required.

4. Population Sharpening Example

This example uses a USGS 1:250,000-scale LU/LC map to sharpen county population data in and around Albuquerque, New Mexico. Fig. 1a shows the LU/LC map with county census data overlaid. Fig. 1b is the output of the population data sharpening process. The population densities within the four counties estimated from the sharpened data are in reasonable agreement with the original data.

Once the population densities of the map classes have been estimated, population maps can be computed outside of the training area. Fig. 2 shows the results for the city of Albuquerque. When this experiment was performed, the population of Albuquerque was 379,300 people. The population estimated by summing the density estimate within the city limits was 395,152 (within 5%).

Going further we can use imagery instead of USGS LU/LC data to compute population maps. Fig. 3 is a full Landsat TM scene (9000 x 8530 pixels, at 25 m/pixel) and a population map computed from a spectral class map (not shown).

5. Summary

Spatial sharpening techniques that fuse images and maps were reviewed. A new method of spatially enhancing and extending limited population data over large areas using maps

---

2 See, for example http://www.umiacs.umd.edu/research/GC/mix_mod/eqns/node3.html
and/or imagery was described. Additional enhancements may further improve its flexibility and accuracy including use of spatial information (e.g., size, shape, and context), and fusion with other data sources (e.g., slope/aspect).

Fig. 2 Extended result over Albuquerque, NM

Fig. 3 Computing a population map over a full Landsat scene

References